Considerations on Harmonic Impedance Estimation in Low Voltage Networks

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***Abstract*—One difficulty in calculating harmonic voltages and currents throughout a transmission or distribution system is the need for a precise model of the linear load, both in magnitude and composition, fed from each bus. It has become evident that the use of equivalents without a comprehensive check on the effects of all impedances actually present can lead to inaccurate estimation of the harmonic voltages and currents. Considerations on harmonic impedance estimation in low voltage networks are presented in the paper. Influences of model abstractions and uncertainties in parameter estimations are analyzed analytically and tested on a model of a real low voltage network. The parameters analyzed include different load compositions, cable lengths, lumping loads and feeders, and medium voltage network representations. It is observed that some of these parameter changes have only a minor effect on the frequency of the first parallel resonance, while other effects have to be included in the calculation to avoid misleading results. This analysis can be used as a guideline when harmonic voltages are estimated in low voltage networks.**

***Index Terms*—Impedance, power system harmonics, uncertainty, resonance.**

I. Introduction

HARMONIC analysis is performed for power systems to determine the effect of harmonic sources on the harmonic voltage levels in the system. As the number of non-linear devices (both loads and generators) in low voltage (LV) networks is increasing, these studies are often performed for low voltage distribution systems.

Simulating the impact of one or more harmonic sources faces two difficulties, modeling the source of harmonics and modeling the equivalent system impedance. This paper focuses on the harmonic impedance modeling.

The system impedance is influenced by many elements. Knowing the exact composition of loads, both in the low voltage network and upstream networks, is usually difficult and also changing in time. Network reconfigurations also add to the time varying nature of the impedance, so each calculation can serve only for a particular moment in time. For this reason it is usual to calculate the polar diagram of the impedance for all predicted topology and load changes.

Calculations and analysis of harmonic impedances in transmission systems can be found in [1]-[5]. Different models of system elements are described in [1], [3]-[4], and [6]-[10]. Examples of distribution system impedance modeling are given in [6], [11]-[14]. Sensitivity of impedance estimations are discussed in [2], [4], and [15]-[17].

Uncertainties of impedance modeling in low voltage networks and errors caused by simplifications and parameter errors were not analyzed in previous works. The aim of this paper is to analyze these effects, and to estimate possible errors in harmonic impedance calculations due to different model and parameter changes.

Model changes were analyzed analytically, and on an example of a real low voltage cable network, in which a parallel resonance was observed. This analysis can be used as a guideline in the low voltage modeling process. It emphasizes which parameters of the network have a significant impact on the resulting harmonic impedance, in contrast with parameters which can be simplified with minor errors.

II. Element Models and Example Network

The adopted test network is a household low voltage network with a large amount of PV inverters connected. This network was chosen because the capacitance of PV inverters shifts the first parallel resonance in the low frequency range [18].

Cables were modeled with their PI equivalents. Skin effect was not taken into account. Both the LV and the medium voltage (MV) networks are cable networks.

Transformers were modeled as series *RL* circuits. Their capacitances were not taken into account as the maximal frequency of interest was 3 kHz.

Power factor correction (PFC) units were modeled only as a capacitance without losses.

Household loads were modeled as parallel *RC* circuits and

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parallel *RLC* circuits (several scenarios). The capacitance should represent the input capacitance of all power electronic devices, mainly their input filters. In [13], a range of (0.6 – 6) µF per house is proposed. In this paper 0.6 µF per house is adopted. Induction motors were modeled as their locked rotor inductance, as proposed in [10], [12]. The total adopted power of linear loads in houses was 500 W, and induction motors were accounted as (0 – 30) % of this load, in several steps. Resistance should represent the linear loads without motors. Depending on the amount of induction motors, resistance was changed to get the same total power of linear loads.

Photovoltaic inverters were modeled as their input capacitance. Reference [13] proposes using values of (0.5 – 10) µF for a (1 – 3) kW inverter, based on measurements. In this paper, several values are used, to show the effect if this value is not known. The total installed power of PV inverters in the low voltage network is 300 kW, mostly composed of 2 kW units, while the peak load of all loads together is approximately 150 kVA.

The effect of lumping loads was examined in three steps. In the first step, all loads were connected directly at the low voltage busbar. In the second step, feeders were separated in the low voltage network, with lumped loads on feeders and feeder branches. In the last step, all houses and inverters were modeled separately.

The medium voltage network was modeled in two ways. The simple version of the model is a series *RL* circuit, representing the short-circuit power of the network and the *R* to *X* ratio. A more detailed model was also used, representing all MV feeders until the HV/MV substation, and one 1.4 MW CHP (combined heat and power) generator in the MV network and several configurations of PFC in the MV network. The HV network was represented with its short-circuit level.

A schematic diagram of the low voltage part of the example network is shown in Fig. 1. The medium voltage part of the network is presented in Fig. 2. The low voltage network is connected to busbar 13 of the MV network.

All four feeders are numbered on Fig. 1, while on Fig. 2 only four busbars are numbered (2, 9, 12, and 13), since changes of elements were applied only on these busbars.

III. Analysis of the Harmonic Impedance

The analysis of the network harmonic impedance is divided into several parts. The effect of lumping loads, load models, cable lengths, and MV network models are investigated separately. Mechanisms of parameter changes are examined analytically (for a simplified example), and on a DIgSILENT Power Factory model of the example network, using the frequency scan. In all cases the impedance was observed on the low voltage side of the MV/LV transformer.

A. The effect of lumping loads

The number of loads in a LV network is usually too large to allow for modeling each load separately. For this reason, loads are commonly lumped into equivalent loads with some feeders and load parameters neglected. This leads to uncertainty of the outcome.

To illustrate the effect of lumping analytically, we can look at a simplified network model with two parallel feeders as in Fig. 3 and derive its equivalent impedance. In this figure, L represents the upstream system inductance, L_1 and L_2 represent feeder inductances, and C_1 and C_2 represent capacitances of loads connected to these two feeders.

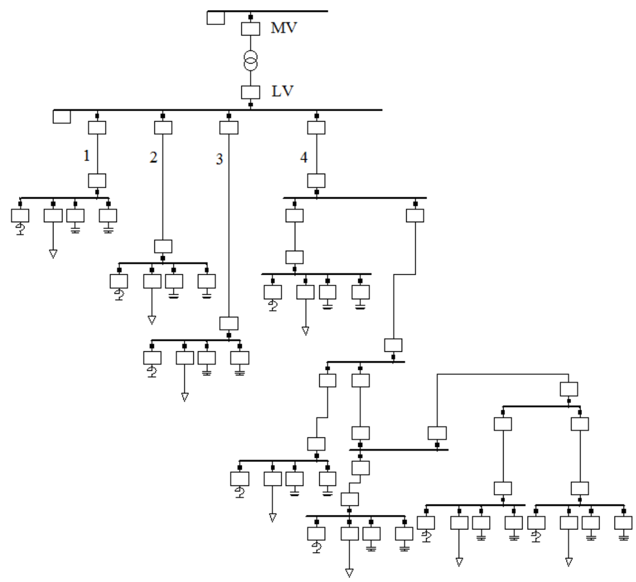

Fig. 1. Low voltage part of the example network

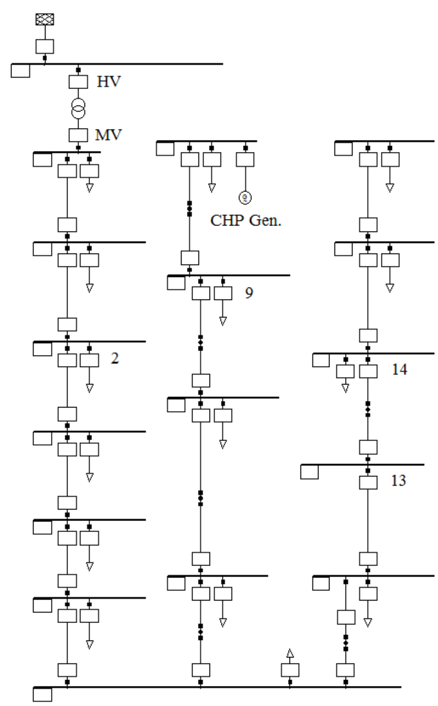

Fig. 2. Medium voltage part of the example network

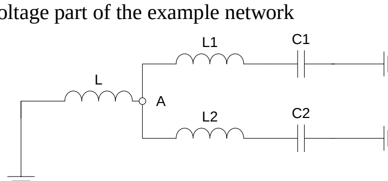

Fig. 3. Simplified example for lumping feeders

If we lump the two capacitances together (as $C_1 + C_2$), and neglect the inductances L_1 and L_2, the impedance Z_A at point A is given by:

$$Z_A = j\omega L/(1 - \omega^2 L(C_1 + C_2)) \quad (1)$$

If we do not lump the two feeders and take inductances L_1 and L_2 into consideration, the impedance is given by:

$$Z_A = \frac{j\omega L(1 - \omega^2 L_1 C_1)(1 - \omega^2 L_2 C_2)}{1 + \omega^4 L_1 L_2 C_1 C_2 - \omega^2 (L(C_1 + C_2) + L_1 C_1 + L_2 C_2)} \quad (2)$$

If the inductance L is much greater then L_1 and L_2, and the lowest parallel resonance frequency f_{r1} is in the low frequency range, equation (2) will give a solution for f_{r1} which is close to the solution of (1). Besides the difference in calculated f_{r1}, L_1 and L_2 also introduce two series resonances and an additional parallel resonance with a higher resonant frequency. The second parallel resonant frequency increases as L_1 and L_2 decrease. If cable lengths are not very long, the resonant frequency of the second parallel resonance and both series resonances are usually too high to be of interest for distribution systems.

To illustrate the difference between (1) and (2), we assume $C_1 = C_2$ and $L_1 = L_2$ and vary the ratio between L and L_1 from 50 to 5. The difference in the lower parallel resonant frequency calculated by (2) and (1) is shown in Fig. 4.

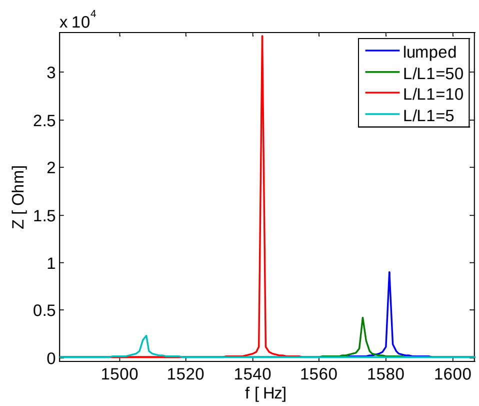

Fig. 4. The first parallel resonance for different element values

The shown uncertainty is dependent on the ratio of impedances. If we know that the feeder inductance is much lower than the upstream system inductance, lumping will not lead to large errors when we calculate the resonant frequency (amplitude will be affected more). However, as feeder length become longer, and this is mostly the case, this error increases.

In a realistic scenario the topology is much more complex than in Fig. 3. To illustrate the effect of lumping on a realistic low voltage network, we compare the harmonic impedance versus frequency at the low voltage busbar of the example network from Fig. 1, for three cases. In the first case we look at the whole low voltage network as a single parallel *RLC* load connected directly at the transformer (case: all lumped). In the second step, we lump the separate feeders as shown in Fig. 1, with feeders and feeder branches lumped as parallel *RLC* loads after cables (case: lumped feeders). In the third step, we uncouple the loads to more branches, with short feeders divided in five sections, and longer feeders in 10 sections (case: no lumping). Results are presented in Fig. 5.

The solution of the "most realistic" case (no lumping) falls between the two other cases. In comparison with the case with everything lumped at the busbar, lumping complete feeders will add extra inductance in the circuit, resulting with a lower resonant frequency (in this case almost 30 Hz). In the case where nothing is lumped, most capacitances are connected via a lower inductance, resulting in a smaller frequency change from the first case (less than 20 Hz).

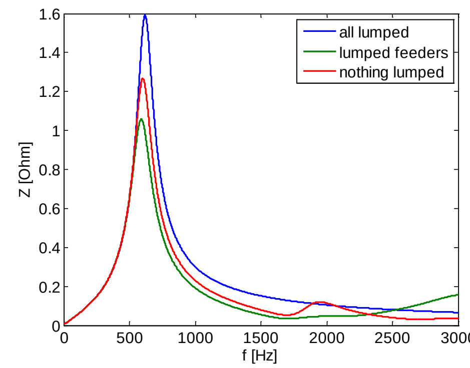

Fig. 5. Effects of lumping loads on the example network impedance

In conclusion, lumping all loads leads to an increase of the resonant frequency, but with acceptable errors if feeder lengths are short. It does not reveal all resonances in the system. Lumping separate feeders leads to a decrease in the resonant frequency, with smaller errors. It also reveals additional resonances but the uncertainty is larger at higher frequencies.

To avoid high model complexity, in the following subsections the model with lumped feeders is used for analyzing other effects.

B. The effect of different load models

Determining appropriate load models is very important and difficult at the same time. Loads are the main damping element in the network, but can also change the resonant conditions, especially at higher frequencies [12].

Load demand and composition change in time, making it necessary to assess several scenarios. Basic assumptions for modeling loads were proposed in [12]:

- Distribution lines and cables should be represented by an equivalent π. The capacitance of lines should be included.
- Transformers should be represented by an equivalent element. Series *RL* circuits are proposed for low frequencies.
- For rotating machined the active power does not represent the damping value, so the active and reactive power demand at the fundamental frequency may not be used straightforwardly. A locked-rotor reactance or a parallel *RL* circuit are proposed.
- Power factor correction capacitance should be estimated as accurately as possible.
- Other elements, such as line inductors, filters and generators should be represented according to their actual configuration and composition.

- The active power of electronic loads should not be taken into account as a resistance. Power electronics should be taken into account as input impedance, e.g. their filter capacitance, or as an open circuit if their input impedance is too high.

Measured active and reactive powers should not be used directly to determine values of linear elements. Active power of power electronic and motor loads should not be included in the value of the resistance. Measured reactive power does not reveal the mixture of inductive and capacitive loads, and short-circuit inductance of motors is lower than the one calculated from reactive power. Finally, the measured reactive power usually contains the distortion power, which does not correspond to physical inductances or capacitances.

To analyze the impact of motor loads, we start with a simplified model from Fig. 6. In this figure, L is the inductance of the upstream system, C is the capacitance of loads, and L_m is the locked-rotor inductance of motor loads.

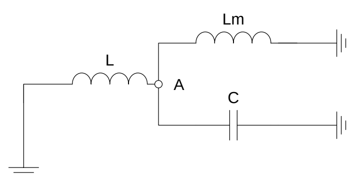

Fig. 6. Simplified example for motor load impact

In this case, the impedance at point A is given by:

$$Z_A = j\omega L_{par} / \left(1 - \omega^2 L_{par} C\right) \qquad (3)$$

where L_{par} is an inductance equal to the value of L in parallel with L_m. Its value is lower than the lower of the two inductances, and if L_m is much higher than L (motor of low power), it will be almost identical to L. As we decrease L_m (increase the power of the motor) the resonant frequency will become higher. If the power of the motor is very large, the resonant frequency would be affected more by the motor inductance.

In a realistic network this dependency becomes more complicated. To illustrate it on the example network, we look at the impedance at the low voltage busbar for different motor load shares in the network – see Fig. 7.

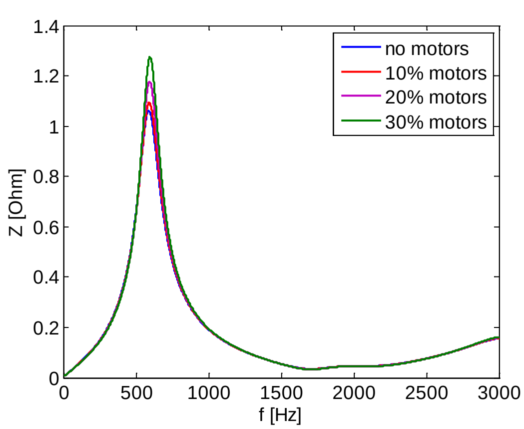

Fig. 7. The effect of motor load share on the impedance characteristic

In the first iteration, no motors were added in the network, and then in several steps the motor share was increased up to 30 % of the active power consumption. These changes result in the changes of the resonant peak (mostly due to the change of resistance), but the resonant frequency is shifted up for only 5 Hz. If the short-circuit power would be lower, the upstream network would have a larger inductance leading to larger differences. This leads to the conclusion that if the amount of small motors is not known in the network, it should not lead to significant errors when determining the lowest resonant frequency. However, neglecting a large motor would lead to larger differences.

To analyze the impact of capacitive loads, we can look at expression (3) again. Capacitances change the resonant frequency directly, a ΔC change of capacitance changes the resonant frequency by $1/\sqrt{\Delta C}$.

In the example network there are no PFC units in the low voltage network, the capacitances are mostly located in input filters of PV inverters. If the value of this capacitances is not known, this leads to a large range of possible solutions. Fig. 8 shows the impedance characteristic for four capacitance assumptions. Initially, 8 µF is assumed for each 2 kW inverter; then a ± 20 % capacitance uncertainty is taken into account; in the end, it was assumed that 2 µF is the input capacitance of each 2 kW inverter.

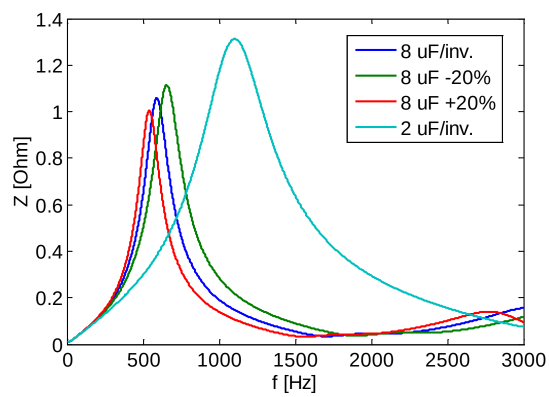

Fig. 8. The effect of capacitance on the impedance characteristic

If the capacitance is not known initially, the difference between assuming 2 and 8 µF per inverter in this case leads to a 500 Hz difference in the resonant frequency. If the capacitance is known, and the uncertainty is taken into account as ± 20 %, differences of 60 Hz can be noticed.

The frequency of the lowest parallel resonance is usually not influenced by the value of resistive loads used. However, resistive loads provide damping in the system, so they are also responsible for peak values of impedance. If we neglect resistive loads, it may happen that we overestimate harmonic voltages due to unrealistic values of impedance. In contrast, if we overestimate the power of resistive loads, we may underestimate the impedance and harmonic voltages.

Fig. 9 shows the effect of variable load resistance in the example network. The initial resistive load was changed for ± 20 %, without changes in other parameters.

The value of the resonant peak changed by about 20 % for both changes, while the resonant frequency changed for only 1

Hz. This is a convenient property of resistance modeling: changing its value will not affect the resonant frequency significantly, and the critical case is always the lowest resistive load (highest resistance). While other parameters need to be analyzed in several conditions, for the resistance it is usually sufficient to observe the case with the lowest resistive loading. As mentioned earlier, this loading should exclude power electronic devices and electrical motors. Also, if higher frequencies are of interest, resistances have a more complicated impact.

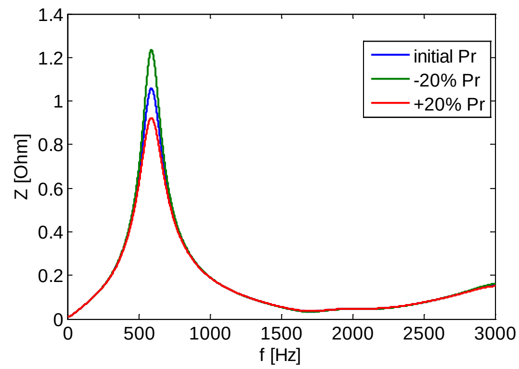

Fig. 9. The effect of resistive loads on the impedance characteristic

C. The effect of cable lengths

The effect of cable lengths was already mentioned in the analysis of lumping loads. It was noticed that complete load lumping with neglected low voltage feeders introduces errors in the resonance analysis, both for the frequency and peak amplitude. In this subsection we analyze the effect of cable length uncertainty on a model with loads lumped on feeders.

Equation (2) can be used to explain this effect. Inductances L_1 and L_2 carry the uncertainty of cable lengths. If these inductances are comparable with L, the final result will be influenced significantly. If they are much lower than L, the effect will not be significant. Fig. 10 shows the effect of ± 20 % cable length changes in the example network. In this example, 20 % changes lead to 5 Hz changes in the resonant frequency. Peaks of the impedance change approximately by 10 %.

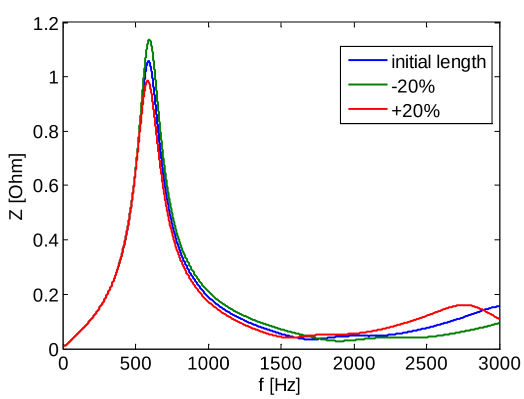

Fig. 10. The effect of low voltage cable lengths on the impedance

D. The effect of MV network representations

In the first approximation, the MV network is usually considered just as its short-circuit impedance. However, some elements of the MV network may be very important for impedance estimation, especially PFC units and generators. Since the impedance of MV network elements is transposed to the low voltage level, approximations in the MV network are similar as approximations on one of the low voltage feeders.

To illustrate the effect of different MV network representations, several topology variations were introduced to the network from Fig. 2: the CHP generator was switched on and off, PFC of 0.5 MVAr was connected to busbars 2, 9, and 14, and PFC of 1 MVAr was connected to busbar 14 directly and through a MV/LV transformer.

In Fig. 11 the model with a single impedance MV network is compared with the model from Fig. 2, with and without the CHP generator connected.

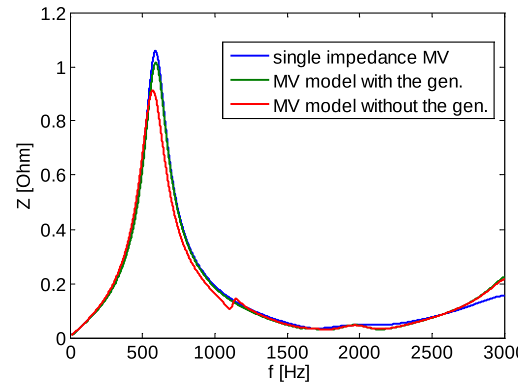

Fig. 11. MV network as a single impedance, and with MV feeders and busbars

In these three cases the fist parallel resonance is being shifted by 20 Hz. Also, the peak amplitude is changing by 15 %, and additional resonances with small peaks are visible with the more complex model.

In Fig. 12 the effect of PFC in the MV network is considered, with and without the CHP generator connected.

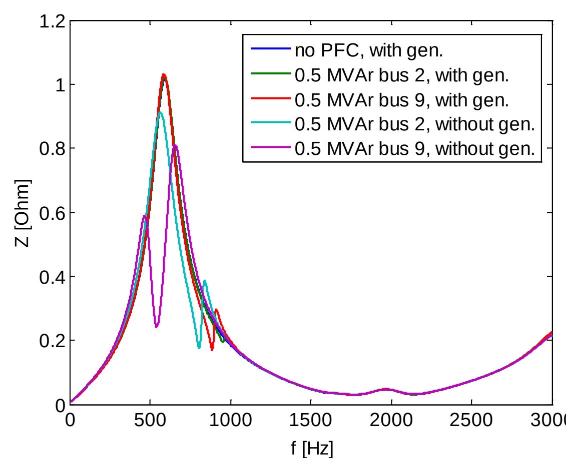

Fig. 12. The effect of PFC in the MV network, busbars 2 and 9

On busbars 2 and 9, 0.5 MVAr PFC causes only minor effects if the generator is connected (resonant frequency reduced for 5 Hz, additional smaller resonance). However, if the generator is disconnected, the effect is much larger.

In Fig. 13 the effect of PFC on busbar 14, which is nearer to the observed LV network than busbars 2 and 9, is considered. It is visible that the effect is more significant if the PFC is nearer the busbar of interest. Also, similarly as with long MV feeders, PFC on other LV busbars has only a minor effect due to the extra MV/LV transformer in between. Generators in the MV network make this interaction more complex, since additional series resonances can create additional impedance peaks near the parallel resonance.

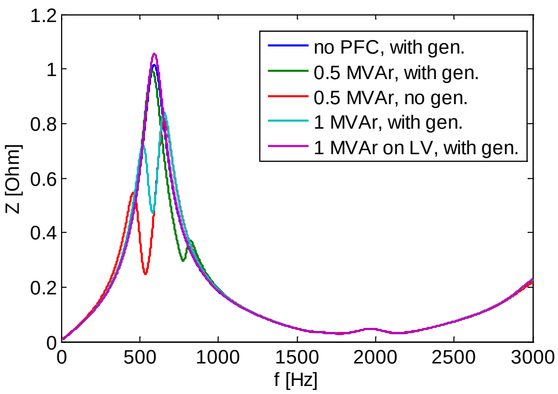

Fig. 13. The effect of PFC on busbar 14 of the MV network

IV. Conclusions

Considerations on harmonic impedance estimation in low voltage networks are presented in the paper. Effects of different model abstractions, linear load compositions, and parameter changes were analyzed analytically and illustrated on an example low voltage network.

The results of the analysis can be summarized as follows:

- Lumping of low voltage loads in some cases introduces large differences in the network impedance. For short feeders in weak grids this effect is not significant.
- The uncertainty of capacitance of loads in the low voltage network has the most significant impact. If this value has to be assumed, large errors should be expected.
- Resistive loads have a minor impact on the frequency of the first parallel resonance. Due to the damping, resistances are important to determine the resonant peak, but the frequency can be determined even with the first approximation.
- The share of motor loads has a significant impact only if the motor locked-rotor inductance is comparable to the upstream system inductance.
- Values of load resistances, inductances, and capacitances should not be derived directly from power measurements. Resistances should be derived from the active power without power electronic and motor loads. Motors should be represented by their locked-rotor equivalents. Capacitances should account only for physical capacitances in the network.
- Cable length assumptions have a significant impact only for long feeders. In other cases, small deviations of cable lengths do not lead to large result changes.
- MV network representations and reconfigurations are very important if power factor correction units are connected directly on the MV level, and/or if generators are present. PFC units in nearby low voltage networks have only a minor impact.

This analysis can be used as a guideline for low voltage harmonic analysis. Choosing appropriate load models and network representations is a vital step for determining harmonic voltages in the network.

V. References

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