Evaluation of Reactive Power Support Interactions Among PV Systems Using Sensitivity Analysis

Afshin Samadi, Robert Eriksson and Lennart Söder

KTH Royal Institute of Technology
School of Electrical Engineering
Department of Electric Power Systems
Stockholm, Sweden 10044
Email: afshin.samadi@ee.kth.se, robert.eriksson@ee.kth.se, lennart.soder@ee.kth.se

Abstract—Growing trends in generating power from distributed PhotoVoltaic (PV) systems has accommodated more and more PV systems within load pockets in distribution grid. This high penetration has brought about new challenges such as voltage profile violation, reverse load flow and etc. A few remedies have been imposed by grid codes such as reactive power contribution of PV systems and active power curtailment. This study applies two analytical methods from control science to find the possibility of controllability among the PV systems in a distribution grid for voltage profile control at specific set-points through reactive power regulation and active power curtailment. For this purpose, the voltage sensitivity matrix is used as the steady-state gain of the multi-variable system. The first method is Relative Gain Array (RGA), in which RGA of the voltage sensitivity matrix is utilized as a quantitative measure to address controllability and the level of voltage control interaction among PV systems. The second method is Condition Number (CN), in which Singular Value Decomposition (SVD) of the voltage sensitivity matrix is used as a mathematical measure to indicate the voltage control directionality among PV systems. Two radial test distribution grids with different feeder R/X ratio, overhead line and underground cable, are employed as

PV systems in a distribution grid would be affected by the extent that improper set-points may lead to interaction among PV systems in the same vicinity. In [11], determinant of voltage sensitivity matrix from load flow calculation has been employed to study the impact of the R/X ratio on the effectiveness of using active and reactive power for regulating voltage profile. In [9], sensitivity matrix has also been used to show the difference between a system with overhead line and underground cable. However, the level of interaction and directionality among the PV systems regarding voltage control to specific set-points has not been addressed in the previous literature.

The aim of this paper is to address the possibility of controllability among PV systems for voltage profile regulation to specific set-points via two analytical control methods. For this investigation, the voltage sensitivity matrix, which can be derived via the load flow calculation, is used as the steady-state gain of the understudy system. The first method is Relative Gain Array (RGA) [12], [13] that is employed to analyze and evaluate the controllability and level of voltage control interaction among the PV systems. The second method is Condition Number (CN), in which mathematical measure of directionality is provided by Singular Value Decomposition (SVD). This method is a useful way to quantify how the range of possible gains of a multi-variable process varies for an input direction [13], [14]. Wide (or narrow) range of possible gains for a process implies large (or small) directionality.

Sub-matrices of the voltage sensitivity matrix indicate the sensitivity of the bus voltages and angels to the variation of active and reactive power at buses. The RGA and CN of the voltage sensitivity sub-matrices, in turn, indicate the degree of the interaction and directionality, respectively. The relation of feeder R/X ratio and the distance between buses in a distribution grid for voltage control is of concern. Applying the aforementioned methods provides an analytical view that how the voltage control interaction and directionality among PV systems in a distribution grid would be affected by the distance and R/X variation.

Two radial test distribution grids with different feeder R/X ratio, overhead line and underground cable, are employed as
the test platform. MATLAB environment is used to calculate the voltage sensitivity matrix and investigate it further via RGA and CN. Derived results, in conclusion, demonstrate decentralized voltage control to specific set-points through the PV systems in the distribution grid is fundamentally impossible due to the high level voltage control interaction and directionality among the PV systems.

In the following, a general overview of the voltage sensitivity will be given in section 2, basic of RGA and condition number are presented in section 3 and section 4 respectively, section 5 presents the simulation platform and section 6 deals with the results and finally the conclusion comes at section 7.

II. VOLTAGE SENSITIVITY MATRIX

Voltage Sensitivity matrix is a measure to quantify the sensitivity of bus voltages (|V|) and bus angles (θ) with respect to injected active and reactive power for each bus except slack bus. Sensitivity matrix is obtained through partial derivative of load flow equations, \( g(|V, \theta) \), as follows [15]:

\[
\begin{bmatrix}
\Delta \theta \\
\Delta |V|
\end{bmatrix} = \left[ \begin{bmatrix}
\frac{\partial g_p(\theta, |V|)}{\partial \theta} & \frac{\partial g_q(\theta, |V|)}{\partial \theta} \\
\frac{\partial g_p(\theta, |V|)}{\partial |V|} & \frac{\partial g_q(\theta, |V|)}{\partial |V|}
\end{bmatrix} \right]^{-1} \begin{bmatrix}
\Delta P \\
\Delta Q
\end{bmatrix}
\]

Voltage sensitivity matrix consists of four sub-matrices that denote the partial derivatives of bus voltage magnitude and angle with respect to active and reactive power. Due to importance of the voltage magnitude regulation by variation of active and reactive power, sub matrices that are related to variation of voltage magnitude, \( S_{|V|,P}^V \) and \( S_{|V|,Q}^V \), are of more interest and concern in this study. Each element of these sub matrices, e.g. \( S_{|V|,P}^V \), is interpreted as the variation that would happen in a voltage at bus \( i \) if the active power (or reactive power) at bus \( j \) changed 1 p.u. Voltage sensitivity matrix represents the open loop gain of the system which is later used as the steady state transfer function of the system to conduct some investigation.

Equation (1) represents a linearized form of the system equations. Keeping this in perspective, it follows from (1) that voltage magnitude variation corresponds to active and reactive power variation and consequently in order to keep the voltage magnitude theoretically constant, following is deducted which can be also employed as a measure to determine the degree of active-reactive power dependency.

\[
\Delta Q = -S_{|V|,Q}^V S_{|V|,P}^{-1} \Delta P = J \Delta P
\]

(2)

Equation (2) is used later to compare the relation between the reactive power and active power while the voltage profile is perfectly controlled.

III. RGA METHOD

Although the RGA was basically introduced by Britsol [12] for pairing the input and output variables in a decentralized control system, it has also been exploited as a general measure of controllability [13], [14]. The relative gain array has been addressed in many literatures and is frequently employed as a quantitative measure of controllability and control loop interaction in decentralized control design. The RGA is originally formulated for steady state analysis and later it was extended to include the dynamics [13]. In this study, the RGA concept is used to analyze the voltage sensitivity matrix, which is calculated from system algebraic equations and therefore does not comprise dynamic.

The proposed interaction measure through RGA indicates how the apparent transfer function between manipulated or input variable (\( u_i \)) and controlled or output variable (\( y_j \)) is affected by control of other controlled variables. This measure is shown by \( \lambda_{ij} \) and is described by the ratio of the transfer function between a given manipulated variable and controlled variable while all other loops are open, and the transfer function between the same variables while all other outputs are closed as follows:

\[
\lambda_{ij} = \left( \frac{\frac{\partial y_j}{\partial u_i}}{\frac{\partial y_j}{\partial y_k}} \right) \bigg|_{u_i \neq \text{constant}} \bigg|_{y_k \neq \text{constant}}
\]

(3)

In other words, the RGA is the ratio of the open loop gain between two variables to the closed loop gain of the same variables while other outputs are perfectly controlled. For a MIMO system with \( G(0) \) as the steady state transfer function, the RGA is attained as follows:

\[
\Lambda(G(0)) = G(0) \times (G(0)^{-1})^T
\]

(4)

Where \( \times \) denotes element-by-element multiplication.

Equation (3) demonstrates that the open loop gain between \( y_j \) and \( u_i \) changes by the factor \( \lambda_{ij}^{-1} \) while the rest of loops are closed by integral feedback control. This implies that the pairing should be preferred for RGAs that are as close to unity as possible. \( \lambda_{ij} = 1 \) implies that there is no interaction with other control loops. Intuitively, decentralized control requires an RGA matrix close to identity [13]. In a decentralized control, the MIMO process works as several independent SISO sub-plants. If RGA elements are greater than one, the decoupling or inverse-based controller can be used to decouple interactions. However, systems with large RGA elements are basically hard to control owing to big interactions and input uncertainties; by doing so, inverse based controller should be prevented since it is not robust. Pairing with negative RGA elements must be avoided because those lead to integral instability.

Sub-matrices of the voltage sensitivity matrix in (1) are steady-state gain of the system and by doing so the RGA of \( S_{|V|,P}^V \) and \( S_{|V|,Q}^V \) are given as follows:

\[
\Lambda(S_{|V|,P}^V) = S_{|V|,P}^V \times \left( S_{|V|,P}^{-1} \right)^T
\]

(5)

\[
\Lambda(S_{|V|,Q}^V) = S_{|V|,Q}^V \times \left( S_{|V|,Q}^{-1} \right)^T
\]

(6)

The RGA of \( S_{|V|,P}^V \) in (5) can be used to study the possibility of controllability and interaction among voltage controllers of PV systems via power curtailing in order to regulate the voltage of buses to specific set-points. The RGA of \( S_{|V|,Q}^V \) in (6) is used to investigate the possibility of controllability and interaction among voltage controllers of PV systems to
regulate voltage of buses to specific set-points via regulating reactive power.

To sum up, in RGA method, the voltage sensitivity matrix must first be derived. Then, RGA of sub-matrices $S_{V,P}^V$ and $S_{V,Q}^V$ are calculated. In the next step RGA values are evaluated. RGA values close to one demonstrate a decentralized system. If the RGA values are big but less than 5, the decoupling compensators can be used to make the system decentralized. However, large RGA values, more than 5, correspond to controllability problems because of big interactions and input uncertainties [13].

IV. CN METHOD

Another measure to quantify the level of interaction in multi-variable systems is condition number. CN of a system is defined as the ratio between maximum and minimum singular values of the system, which are computed using SVD [13], [14]:

$$\gamma(G(0)) = \frac{\sigma(G(0))}{\sigma_1(G(0))}$$

(7)

A process with large CN implies high directionality and is called to be ill-conditioned [13]. The steady state gain of MIMO process varies between $\sigma(G(0))$ and $\sigma_1(G(0))$. Wide range of possible gains for a MIMO system indicates large directionality. Such a plant is often considered sensitive to uncertainty that, in turn, will lead to a poor robust performance [13]. Moreover, a large CN results in control problem. A large CN may be brought about by a small singular value that is generally undesirable.

In a nutshell, in CN method, the voltage sensitivity matrix must first be derived. Then, SVD of sub-matrices $S_{V,P}^V$ and $S_{V,Q}^V$ are computed and consequently CN is calculated. CN larger than 50 demonstrates controllability problems [13].

V. PLATFORM OF THE SIMULATION

Radial grid in Fig. 1, which consists of five houses connected through a step down transformer to a medium voltage grid, is employed as a test grid in this paper. In this study, it is assumed that all the houses have been equipped with PV systems. In this grid both overhead lines and underground cables are taken into consideration in order to study the effect of the R/X ratio. The parameters of the test radial grid have been given in Table I [9].

In the load flow calculation, the slack bus is naturally excluded from sensitivity matrix. Moreover, in the sensitivity matrix, rows and columns corresponding to buses that have no PV systems are also neglected.

| Table I |
|-----------------|-----------------|-----------------|-----------------|-----------------|
| Grid impedance  | 1.4e-4 +1.4e-4i p.u. | Transformer impedance | 0.01035 + 0.00667i p.u. | Over head line impedance per km | 0.0516 + 0.0375i p.u. | Underground cable impedance per km | 0.04000 + 0.01020i p.u. |
| Rated total net load | 20 kW | Base Voltage | 400 V | Base Power | 20 kW |

VI. RESULTS

A. Sensitivity matrix characteristic

Figs. 2 and 3 show the spectrum of the diagonal elements of $S_{V,P}^V$ and $S_{V,Q}^V$ for overhead lines and cables, respectively. As it was expected the sensitivity to reactive power in overhead line is noticeably bigger than underground cable. Nevertheless, in case of underground cable, it can be seen that at the beginning of the feeder, sensitivity to reactive power is higher compared to active power, but as approaching to the end of feeder it gets the other way around. Therefore, even though resistive part of the underground cable is dominant, controlling voltage profile by regulating reactive power at the beginning of the feeder, seems to be more effective.

B. Voltage regulation active-reactive power dependency

Irrespective of the operating point and R/X ratio, (2) yields an upper triangular matrix. Nevertheless, the diagonal elements and first row of the matrix, which are dominant elements, vary significantly between the overhead line and underground cable. Figs. 4 and 5 depict the spectrum of those elements.

The characteristics of the matrix is summarized as follows:

- The first entry in the diagonal and the first row are common and corresponds to the first bus, which can only see the impedance of the grid, and by doing so it gets same value in both systems with overhead line and underground cable.
Figure 3. The sensitivity spectrum of the diagonal elements of $S^V_{V,P}$ and $S^V_{V,Q}$ for underground cable.

Figure 4. Spectrum of diagonal and first row elements of active-reactive power dependency for overhead line.

Figure 5. Spectrum of diagonal elements and first row of active-reactive power dependency for underground cable.

- Diagonal entries, except the first entry, are almost similar; first row entries, except the first entry, are also almost similar.
- The diagonal entries are almost equal to the feeder R/X ratio in both systems, overhead line and underground cable.
- The absolute difference between corresponding diagonal and first row entries, except the common entry, is almost equal to the absolute value of the common entry.
- Large elements in case of underground cables, which is in conjunction with large R/X ratio, implies that for an identical change in active power of buses, required reactive power to keep voltage profile constant varies largely. In other words, the required reactive power to keep voltage differences equal to zero ($\Delta V = 0$), is proportional to the feeder R/X ratio. By doing so, for feeders with R/X ratio more than one the required reactive power difference ($\Delta Q$) at each bus would be greater than the active power difference ($\Delta P$) in the same bus.
- Depending upon the R/X ratio value, the sign of the first row entries except the first entry changes. In order to study the effect of the k=R/X ratio, the total amount of the overhead line impedance is taken into account, and its R/X ratio is varied. It is observed that for k smaller than 0.58 the sign of the first row entries is negative. Therefore, for small R/X ratio, if the active power difference ($\Delta P$) in all buses are in one direction, the reactive power difference ($\Delta Q$) at all buses will be in one direction as well. However, for large k values the sign of the first row entries are positive and opposite of the diagonal entries which means the reactive power variation at bus one is always in contrary with other buses.

Eq. (2) is used to calculate the required reactive power adjustment to compensate the voltage profile fluctuation owing to the variation of active power. Considering the initial operating point at $P_0 = 0$ and $Q_0 = 0$ gives

$$\Delta P = P - P_0 = P$$
$$\Delta Q = Q - Q_0 = Q$$

$$P = JQ \quad (8)$$

Consequently, the needed power factors for the PV connected buses are calculated as follows:

$$PF = \frac{P}{\sqrt{P^2 + (\sum J')^2}} \quad (9)$$

Where PF is a vector consisting of power factors at each PV installed bus. Fig. 6 depicts the power factor of each bus for different R/X ratio while it is assumed that the total net power at each bus has been changed 1 p.u. ($P=1$ p.u.), as can be seen the required power factor varies drastically by increasing R/X ratio. It boils down to this fact that required reactive power to compensate voltage fluctuation depends upon R/X ratio.
C. RGA

Subsequent to the previous section upshot, if adequate reactive power can be provided by PV systems, this question is raised whether it is possible to regulate the voltage of each bus with installed PV system to a fixed set-point through reactive power regulation or not. In this section and following, the interaction among PV systems in a radial distribution grid is quantified by RGA concept to address the possibility of controllability concerning voltage profile regulation to specific set-points.

The RGA of the $S_{V|P}$ and $S_{V|Q}$ look like a block tridiagonal matrix which positive elements are only located on the diagonal and elements on the upper diagonal and on the lower diagonal are negative. According to the RGA pairing rule, therefore, the elements on the diagonal must be paired. This block tridiagonal shape of the RGA of voltage sensitivity sub-matrices indicate that open loop gain of the system, which is the sensitivity matrix, is changed with positive sign on the diagonal and with negative sign on the upper diagonal and lower diagonal. Moreover, since the other elements of the RGA are almost zero, open loop gain of the system on these positions are changed with infinite factor which means these loops are considerably affected by other loops. Figs. 7 and 8 depict the diagonal entries spectrum of RGA of $S_{V|P}$ and $S_{V|Q}$ for overhead lines and cables while all buses are on full production, respectively. It can be seen by moving towards end of the feeder, except the last bus, the level of interaction is increasing. Since the last bus at the end of feeder is affected only by one previous neighbor bus, the level of interaction drops at this bus.

Concerning overhead line, Figs. 9 and 10 demonstrate maximum RGA of $S_{V|P}$ and $S_{V|Q}$ for different net load levels and different line distances between buses. One sees that the interaction level decreases by increasing the distance between the buses, or in turn by increasing the impedance. Moreover, it can be seen that the maximum RGA of $S_{V|P}$ declines by shifting total net load from consumption to production. Similar results, not shown here, are derived for under ground cable.

Figs. 11 and 12 show the impact of the lagging and leading power factor on the maximum RGA of $S_{V|P}$ and $S_{V|Q}$ for different loading conditions, while it is assumed that overhead line segment are 70 m. As can be seen the power factor has relatively very small effect on the maximum RGA of $S_{V|P}$ while the maximum RGA of $S_{V|Q}$ slightly increases by lagging power factor and decreases by leading power factor. The performance of the system with underground cable, not shown here, is analogous with overhead line.

The results of the maximum RGA for different k=R/X are shown in Figs. 13 and 14. It is assumed that the distance between buses are 70 m and power factor is unity. It is obvious that maximum RGA of $S_{V|P}$ increases for larger k values. It is, therefore, deduced that increasing R/X ratio would boost the interaction level among voltage controllers of PV systems regarding reactive power regulation. However, it can be seen in Fig. 13 that the maximum RGA of $S_{V|P}$ declines by large k values.

Based on the depicted results, the positive elements of the RGA of $S_{V|Q}$ are always much bigger than one irrespective of the R/X ratio, total net load and power factor. It can be, therefore, concluded that it is not possible to have decentralized voltage control in order to regulate voltage to a specific set-point at each bus even for small R/X ratio that technically adequate reactive power can be produced by PV systems [13]. Since the RGA of $S_{V|P}$ are much bigger than one, decentralized control based on the power curtailing is not also possible.

Furthermore, the results demonstrate that maximum positive elements of RGA of the voltage sensitivity matrix are large, more than 5, by doing so using decoupling controllers, in order to make a decentralized system, can fundamentally lead to control problems due to sensitivity to inputs [13]. Thus, inverse-based controllers must be avoided.

D. Condition number

At production net load level with unity power factor, CN of $S_{V|P}$ and $S_{V|Q}$ for overhead line are $\gamma_{P,OHL}=44.2$ and $\gamma_{Q,OHL}=72.1$, and for underground cable are $\gamma_{P,UGC}=50.8$ and $\gamma_{Q,UGC}=197.2$. These CNs denote that sensitivity matrix is
Figure 8. The RGA spectrum of the diagonal elements of $S_{V,P}^V$ and $S_{V,Q}^V$ under an underground cable line.

Figure 9. Maximum RGA entry of $S_{V,P}^V$ for different net load levels and different distances between buses, overhead line.

Figure 10. Maximum RGA entry of $S_{V,Q}^V$ for different net load levels and different distances between buses, overhead line.

Ill-conditioned and the severe case is for $S_{V,Q}^V$. Figs. 15 and 16 illustrate the spectrum of the singular values of $S_{V,P}^V$ and $S_{V,Q}^V$, respectively. As can be seen the sensitivity matrix in both systems suffers from high directionality.
Furthermore, smallest singular value for $S_{V|Q}^V$ in the system with the underground cable is smaller than the system with overhead line that implies more directionality and more control problems. These results are in conjunction with RGA results.

Figs. 17 and 18 demonstrate the condition numbers of $S_{V|P}^V$ and $S_{V|Q}^V$ for different R/X ratio and different total net load levels. Regarding $S_{V|Q}^V$ the more increasing $k$ the further CN goes that is along with the RGA results. Analogous with the RGA results, large R/X ratio results in relatively smaller CN for $S_{V|P}^V$. Changing power factor and the distance between buses yield similar results, not shown here, for CN as the RGA results in the previous section.

**VII. Conclusion**

This paper applies two analytical control methods, namely Relative Gain Array and Condition Number, to voltage sensitivity matrix in order to find the possibility of the controllability. RGA and CN are used to quantify the level of interaction and directionality among PV systems in distribution grids regarding voltage control, respectively. The sensitivity matrix is used as the steady-state gain of the system in this study. Moreover, the characteristic of the sensitivity matrix is employed to show the level of dependency
of reactive power to active power for voltage control. The results show that decentralized voltage control to specific set-points through reactive power regulation or active power curtailing is not possible due to large RGA elements and large CN of voltage sensitivity matrix. It is, furthermore, shown that using decoupling controllers to make system decentralized must also be avoided on the grounds that the RGA elements of the voltage sensitivity matrix are too big, larger than 5, that would result in poor control performance.

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References


